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The calculation method of hydrogeological parameters from unsteady pumping test with circular constant water head boundary of finite scale

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Abstract: Laboratory model experiments can be used to verify the applicability and accuracy of a new hydrogeological test method. Unsteady pumping test is the most commonly used comparison method. The axisymmetric circular aquifer model is frequently used for laboratory model experiments. However, the scale of the laboratory experiment model is always finite, which brings great influence on the test calculation results when using conventional Theis theory. Here, on the basis of the analytical analysis and laboratory experiments, a new calculation method is proposed for the circular constant water head boundary of finite scale. The proposed method can eliminate the effect of circular constant water head boundary of finite scale on unsteady pumping test results and also be applied to large-scale circular constant head boundary. Geotextile has the characteristics of permeability. Therefore, the hydraulic conductivity of geotextile is used to simulate the hydraulic conductivity of a fractured aquifer. Geotextile test, the steady pumping test and slug test are conducted to verify the accuracy of the proposed method, all indicating that the proposed method can provide more accurate hydrogeological parameters from an unsteady pumping test with circular constant water head boundary of finite scale than conventional Theis theory of the unsteady pumping test.

Keywords
Unsteady pumping test, Finite scale, Circular boundary, Constant water head, Laboratory model experiment, Hydrogeological parameters

Hydrogeological parameters are quite important for the study of groundwater movement in rock and soil, which involve many engineering fields, such as water conservancy and hydropower engineering, mining engineering, nuclear engineering, municipal engineering and traffic engineering foundation, as well as groundwater pollution and remediation technology (Xue et al. 1997). In the field of hydrogeology, a lot of methods have been put forward to determine hydrogeological parameters better, faster and more accurately (Wang et al. 2018b). A pumping test is one of the most frequently used techniques to verify the applicability of other new test methods (Sakr 2001; Weber and Chapuis 2013; Paradis et al. 2016). Laboratory tests are conducted to verify the applicability and accuracy of a new method to determine hydrogeological parameters. To evaluate the
applicability and accuracy of the new method, it is an essential part to compare the results of the new method with those of the pumping test.

Many researchers have made efforts in the way to determine hydrogeological parameters. Kruseman et al. (1994) proposed pumping tests have proved to be one of the most effective ways of obtaining hydrogeological parameters. Carr and Van Der Kamp (1969), Ihan et al. (2003) proposed the tidal method to determine the hydraulic conductivity and compared it with the pumping test. Ratnam et al. (2005) presented a new permeability parameter test method which needs to be compared with the pumping test method. Zheng et al. (2005) put forward an improved straight-line fitting method for analyzing pumping test recovery data. Eid (2007) conducted a comparative experiment between the empirical formula, deduced from evaluating rock permeability based on core acquisition rate and fracture filling, and the pumping test, in order to verify the applicability and validity of the deduced formula and technology. Cheong et al. (2008) compared hydraulic conductivity derived from a numerical model with those derived from the grain-size methods, slug tests and pumping tests to determine the degree of deviation from the numerical model. Zhou et al. (2016) proposed a new on-site rapid test system to determine the permeability parameters of rock and soil. To verify the reliability and stability of the newly developed testing system, an indoor test platform was constructed. Jazaei et al. (2016) came up with a novel approach that the concept of mean action time was used to predict aquifer response time scales in a two-dimensional radial geometry for pumping, injection and recovery processes. These predictions were verified in a laboratory scale aquifer. The pumping test in laboratory model is still a significant method to study the new theory in the hydrogeological field (Taylor and Howard 2000; van der Kamp 2001; Sun 2015; Ahmed et al. 2016; Li et al. 2018; Zhao et al. 2019).

Many parameters (e.g., fracture width, roughness etc.) have a great influence on the hydraulic conductivity of micro fractures (Frampton and Cvetkovic 2010; Cvetkovic and Frampton 2012; Wu et al. 2013; Cvetkovic et al. 2020; Zou and Cvetkovic 2020). This paper focuses on the hydraulic conductivity of fractured confined aquifer, thus without regard to the influence of fracture width and roughness on hydraulic conductivity of fractured confined aquifer system.

For general laboratory models, an axisymmetric circular aquifer model with the pumping well at the center is usually constructed, where the groundwater well flow can be treated as the radial axisymmetric flow (Singh 2001; Lin et al. 2016; Lee et al. 2018). In that case, the influence of boundary on the test results has to be considered in
pumping test due to the finite scale of the aquifer model. A lot of researches have put emphasis on the affection of linear boundary, but only a few on the non-linear boundary e.g., fan shaped (Chapuis et al. 2006; Firmani et al. 2006; Christensen et al. 2010; Weber and Chapuis 2013). There is no known literature specifically focusing on the case of the aquifer with a circular boundary. The problem of circular boundary can only be solved by steady flow of Dupuit formula (Yang et al. 1980; Xue et al. 1997; Lee et al. 2018; Wang et al. 2018b). However, it is always time-consuming to reach a steady flow state for the pumping test. Moreover, only the transmissivity coefficient can be calculated by the Dupuit formula, while another important hydrogeological parameter, storage coefficient of aquifer, cannot be obtained (Zhao and Zhou 2010; Rabinovich et al. 2015; Su et al. 2015). The unsteady flow theory is frequently used to calculate the parameters in the pumping test, which can obtain both storage coefficient and transmissivity coefficient (Yang et al. 1980; Yang and Yeh 2007; Roques et al. 2014). Nevertheless, in the application of unsteady flow Theis theory to parameter calculation, an important premise should be assumed that the aquifer extends infinitely (Karasaki et al. 1988; Lee and Lee 1999; Mace 1999; Chen 2008). It is difficult for a laboratory model to satisfy such a condition, as it is usually with a finite scale. Inaccurate comparative data will be obtained, if the conventional unsteady flow Theis theoretical formulas are directly applied to calculate the parameters with the unsteady flow pumping test data from a finite scale laboratory model.

Whereas, circular constant water head boundary of finite scale will affect the test results of the pumping test in the indoor circular model or circular island terrain. There are errors in the calculation of hydraulic conductivity by conventional unsteady flow Theis theory. Here, only the constant water head boundary problem is studied, since the variable water head boundary problem affected by tide is not involved. Hence, the influence of tides does not need to be considered. A new method is proposed, which can remove the influence of circular constant water head boundary of finite scale on the pumping test results. In view of that, this paper presents a novel calculation method of hydrogeological parameters from unsteady pumping test with circular constant water head boundary of finite scale. The standard curve of constant flow rate unsteady flow pumping test and calculation procedures are introduced in detail. Different hydrogeological tests in the laboratory scale are conducted to verify the accuracy and practicability of the proposed method, including geotextile test, pumping test and slug test.

**Laboratory model experiment prototype**

A circular fracture aquifer system with horizontal fracture is constructed as the physical model. Here, this model has two major components. The first one, 3.4 m in diameter and 0.28 m in height, is located in the center of a
circular flume with an inner diameter 4 m. Two pressure boxes are installed in the cylindrical base to measure the overlying load. Piezometric tubes are installed around the pipe well set at the center with an inner diameter of 80 mm. The second one is with the same diameter as the first and a height of 0.55 m, located on the top of the first. Between those two components, 5 mm thickness geotextile with a plastic film on the top is placed to simulate the hydraulic conductivity of the horizontal fracture with fillings. The first component simulates the impermeable bottom plate of the fractured confined aquifer, and the second component simulates the impermeable top plate of the fractured confined aquifer. Overflow ports are arranged above the ground with a height of 1.58 m and 1.88 m to simulate the constant water head boundary at different water head heights. The physical model is shown in Figure 1. The #1, #2, #3, #4, #5, #6, #7, #8, #9 and #10 piezometric tube are 168cm, 29.5cm, 53.5cm, 92.5cm, 131cm, 126.5cm, 90.5cm, 50.5cm, 25.5cm, 168cm away from the model center, respectively.

The indoor test model for pumping test is conducted step by step. Firstly, open the water injection valve and the overflow port. The water injection valve is used as the water supply device. The overflow port is to maintain the constant water head boundary and keep the model water level stable. Then, open the pumping valve. The pumping valve is connected to the pipe well, which is used to simulate the pumping of pump. The pumping of constant flow rate under the test conditions can be controlled by the pumping volume. Meanwhile, data of pipe well and piezometric tubes are recorded respectively. The hydraulic conductivity of aquifer can be calculated with the data of pipe well and piezometric tubes based on the steady flow theory and unsteady flow theory (including the conventional unsteady flow Theis theory and the proposed method of unsteady pumping test theory with circular constant water head boundary of finite scale).

In addition, slug test is to excite the water level change (water injection or air pressure excitation) in the pipe well, and then the aquifer hydraulic conductivity is calculated according to the slug test theory based on the time curve of water level recovery in the pipe well.

**Analytical analysis**

Conventional unsteady flow Theis formula is well applied to calculate hydraulic conductivity, which assumes that the aquifer extends infinitely. However, a finite size circular aquifer model with constant water head boundary is usually adopted to study the hydrogeological parameters test method in laboratory. The laboratory unsteady pumping test will be affected by the boundary. According to the constant flow rate unsteady flow formula in a fully penetrating well in confined aquifer with the circular constant water head boundary (Yang et al. 1980):

\[
 s = \frac{Q}{2\pi K_m} \left[ 2 \sum_{r} \frac{J_0\left(\frac{r}{b}\right)}{P} J_0\left(\frac{P}{P}\right) - 2 \sum_{r} \frac{J_1\left(\frac{r}{b}\right)}{P^2} J_0\left(\frac{P}{P}\right) \right]
\]  

(1)
Where, $J_0$ and $J_1$ are the first kind of Bessel functions of order 0 and 1, respectively; $P$ are all positive roots of $J_0(P) = 0$; $b$ is the radius of the circular constant water head boundary [L]; $m$ is the thickness of aquifer [L]; $r$ is the radial distance from the pumping well center [L]; $s$ is the water level drawdown at the radial distance $r$ from the pumping well center [L]; $t$ is the time of pumping well [T]; $Q$ is the flow rate of the pumping well [$L^3T^{-1}$]; $K$ is the permeability coefficient of the aquifer [$L^2T^{-1}$]; $a$ is the pressure conductivity coefficient [$L^2T^{-1}$].

Due to (Yang et al. 1980):

$$2 \sum_{r} \frac{1}{P^2} \frac{J_0 \left( \frac{r}{b} P \right)}{J_0(P)} = \ln \frac{b}{r}$$

(2)

$$F_j(r, r) = \sum r^j \frac{J_j \left( \frac{r}{b} P \right)}{J_j(P)}$$

(3)

Where, $\overline{r} = \frac{r}{b}$, $\overline{r}_i = \frac{a t}{b}$, the value of $F_j(\overline{r}, \overline{r}_i)$ (Yang et al. 1980) is shown in Table 1.

Here, the permeability coefficient and water storage coefficient still cannot be calculated directly from this theoretical formula. Thus, it is necessary to adopt a certain mathematical transformation and study the method of matching curve to determine the permeability coefficient and water storage coefficient. Draw the standard curve on the base of this theoretical formula (1) after a certain mathematical transformation, and put forward a specific method of using the matching method to calculate the permeability parameters.

Substituting Formula (2) and Formula (3) into Formula (1), the following is obtained:

$$s = \frac{Q}{2\pi Km} \left[ \ln \frac{b}{r} - 2F_j(\overline{r}, \overline{r}_i) \right]$$

(4)

Define:

$$F(\overline{r}, \overline{r}_i) = \frac{b}{r} \sum \Pi \overline{r}_i$$

(5)

$F(\overline{r}, \overline{r}_i)$ can be calculated from equation (5), which are shown in Table 2.

Then:

$$s = \frac{Q}{2\pi Km} F(\overline{r}, \overline{r}_i)$$

(6)

Then take the logarithm of both sides of equation (6), the following is obtained:
Because, \( r_e = \frac{a t}{b^2} = \frac{T e}{S b^2} \), then:

\[
t = \frac{r_s b^2}{T}
\]

(8)

Where, \( T \) is transmissivity coefficient, \( S \) is storage coefficient.

Then take the logarithm of both sides of equation (8), the following is obtained:

\[
\lg t = \lg r_e + \lg \frac{S b^2}{T}
\]

(9)

The second item on the right of equation (7) and (9) are the constants in the same pumping test. The curve of constant flow rate pumping test and the standard curve of \( F(\tau, r_e) \) are in the same shape in a dual-logarithm coordinate system. The difference of two curves lies in the vertical and horizontal coordinates translated the distance of \( \frac{Q}{2\pi Km} \) and \( \frac{S b^2}{T} \), respectively. Hence, the permeability coefficient and the water storage coefficient can be calculated with the corresponding coordinate value of \( \{F(\tau, r_e)\}, \{r_e\}, \{s\}, \{t\} \), and they are taken into the equation \( s = \frac{Q}{2\pi Km} F(\tau, r_e) \) and \( t = \frac{r_s b^2}{T} \), as long as they coincide with the two curves and a matching point is optionally chosen.

Using the proposed method of constant flow rate unsteady flow pumping test in a fully penetrating well in confined aquifer with circular constant water head boundary of finite scale, the procedures for calculating the permeability coefficient and the storage coefficient are as follows: ①Plot the standard curve of \( F(\tau, r_e) - r_e \) on dual-logarithm paper, and the standard curve is shown in Figure 2; ②According to the data of constant flow rate unsteady flow pumping test in a fully penetrating well in confined aquifer with circular constant water head boundary of finite scale, plot the measured curve of \( s - r_e \) on the transparent dual-logarithm paper with the same modulus; ③Place the measured curve on the standard curve, and move it under the condition that keeps the corresponding coordinate axis parallel to each other until the two curves overlap with each other; ④Choose a
matching point randomly and record the corresponding coordinate value of the matching point: \( [F (\overline{r}, r_j)], [r_j], [s] \) and [t], then: 
\[
K = \frac{Q}{2 \pi m [s]} [F (\overline{r}, r_j)] , \quad S = \frac{T}{b' [t]} .
\]

**Experimental verification**

In order to compare the accuracy of determining permeability parameters by different test methods and verify the accuracy of calculating permeability parameters by the proposed method, three different tests, geotextile parameters test, pumping test and slug test, are conducted for comparative study. Based on the indoor model, steady flow pumping test, unsteady flow pumping test and slug test have been carried out, and the comparative analysis of various test results is also conducted. Firstly, the permeability parameters of geotextile are tested to obtain the reference values of the hydraulic conductivity of geotextile, and then the pumping test and slug test are carried out in the laboratory model. The hydraulic conductivity values calculated by the steady flow Dupuit theory and slug test method are used as the benchmark values of the hydraulic conductivity of the laboratory model. Finally, the conventional Theis theory and the proposed new method are used to calculate the hydraulic conductivity based on the unsteady flow data of the pumping test, which is comprehensively compared with the reference values and benchmark values of the hydraulic conductivity.

**GEOTEXTILE PARAMETERS TEST**

The permeability of geotextile varies as its thickness changes. In order to obtain the load borne by the geotextile laid above the model base in the circular horizontal fracture model, two vibrating string pressure boxes numbered 02639 and 02654 are embedded in the model. According to the geotextile parameter test, the frequency value of the pressure box in the circular horizontal fracture model under the test conditions is measured by the vibrating string frequency readout instrument to obtain the hydraulic conductivity of the geotextile under the stress condition of the model. The stress calculation formula of vibrating string pressure box is:

\[
P = k_o (f'_i - f'_o )
\]

In the formula, \( P \) is the stress of the tested pressure box\((kPa)\); \( k_o \) is the sensitivity coefficient of the pressure box\((kPa/Hz^2)\); \( f'_o \) is the initial frequency value of the pressure box\((Hz)\); \( f'_i \) is the working frequency value of the pressure boxes\((Hz)\). The initial frequency and the working frequency values of the pressure boxes are shown in Table 3.

All tests abide by the standard of the People’s Republic of China, Test Methods of Geosynthetics for Highway Engineering (JTJ/T 1999). The test of geotextile thickness and transmissivity coefficient is carried out
step by step on special instruments according to the standard. The thickness of geotextile is measured by thickness gauge. The testing process is shown in Figure 3(a). The thickness variation of the geotextile used in this work under different pressure is measured by thickness gauge, which is shown in Figure 3(b).

The test on horizontal transmissivity coefficients of the applied geotextile is conducted for both parallel and perpendicular of machine directions. The horizontal transmissivity coefficients for both directions can be calculated by the following equation:

$$\theta = \frac{QL}{tBh} \Delta h$$  

(11)

Where, $\theta$ is transmissivity coefficient [L$^2$T$^{-1}$]; $Q$ is volume of flow in the time $t$ [L$^3$]; $L$ is length of geotextile sample along seepage direction [L]; $t$ is the time of measure volume of flow [T]; $B$ is the width of the sample [L]; $\Delta h$ is the water head difference of piezometric tube at both ends of the length $L$ [L].

The test process of horizontal transmissivity coefficients and the horizontal transmissivity coefficients curve of the applied geotextile with stress are shown in Figure 4. According to Figure 4, take $1.4 \times 10^{-5} \text{ m}^3 / \text{s}$ as the value of transmissivity coefficient of geotextile in the horizontal fracture model under test conditions stress level. The geotextile thickness is about 4.62 mm when the stress level is 14.75 kPa based on the curve of Figure 3.

PUMPING TEST

Three groups of pumping test with different conditions are conducted. The flow rates are $7.5 \times 10^{-3} \text{ m}^3 / \text{s}$, $7.0 \times 10^{-3} \text{ m}^3 / \text{s}$, and $7.8 \times 10^{-3} \text{ m}^3 / \text{s}$, and the test numbers were 10000-01, 10000-02 and 10000-03, respectively. According to the specific calculation procedure of the proposed method and Table 2, only when $r/b$ is $1E-6$, 0.1, 0.3 and 0.5 the data of piezometric tubes are calculated. While the indoor test model was constructed, only these piezometric tubes positions, # 3 ($r/b=53.5/170\approx0.3$), # 4 ($r/b=92.5/170\approx0.5$), # 7 ($r/b=90.5/170\approx0.5$) and # 8 ($r/b=50.5/170\approx0.3$) meet this requirement. Piezometric tubes positions of # 1, # 2, # 5, # 6, # 9, # 10 cannot accurately meet the requirements of the matching method of the proposed method. So, the data of 3#, 4#, 7#, and 8# piezometric tubes are taken to calculate the parameters. The pumping test has been repeated for three times under the condition of three different flow rates for three groups, and the test results show high consistency and reproducibility. Therefore, only one set of test data is presented for each flow rate here.

**Steady flow pumping test**
The steady flow Dupuit formula is used to calculate the parameters. The influence radius is 1.7m, which is the distance from the circular constant water head boundary to the pumping well center. The calculation results are shown in Table 4, indicating that the three groups of pumping test from the same piezometric tube data achieve a high level of consistency. The calculated parameters from piezometric tube data are consistent with each other, except that the calculated parameters based on the 8# piezometric tube data are larger than the results from the rest piezometric tubes data. The inconformity might be caused by the high permeability near the 8# piezometric tube during the construction of the test model.

**Unsteady flow pumping test based on conventional Theis theory**

The parameters are calculated based on the Jacob straight linear graphic method of Theis formula of unsteady flow in the fully penetrating well (Xue et al. 1997). The calculation results of the transmissivity and the storage coefficient are shown in Table 5 and Table 6, respectively.

Using the unsteady pumping test data, the transmissivity coefficient calculated by the Jacob straight linear graphic method is compared with that of the indoor geotextile test. Only the transmissivity coefficient based on the data of 3# piezometric tube is similar to that of the geotextile test. The transmissivity coefficient calculated based on the data of 8# piezometric tube is also large, but it is close to the result from the steady flow. However, the transmissivity coefficients based on the data from 4# and 7# piezometric tubes are approximately 2-3 times larger than that of the steady flow and the geotextile test. The calculated results based on the data from 3# and 8# piezometric tube are close to that of the steady flow with a slight increase. The 3# and 8# piezometric tube are 1.165 m and 1.195 m away from the circular constant water head boundary, respectively. The 4# and 7# piezometric tube are 0.775 m and 0.795 m away from the boundary, respectively. The circular constant water head boundary affects the calculation results based on the four piezometric tubes, when using the conventional unsteady flow Theis formula to calculate the transmissivity coefficient. The 4# and 7# piezometric tubes have a greater influence because of the closer distance away from the boundary. The calculated transmissivity coefficients obtained by conventional unsteady flow Theis formula are larger than those of steady flow and the geotextile test. Hence, it is necessary to find out a method to correct the influence of the boundary on the calculation results of the unsteady flow pumping test.

The calculated storage coefficients based on the four piezometric tubes data of the three groups tests are basically the same. The results based on the 8# piezometric tube data is slightly larger, which is also related to the high permeability at the location of 8# piezometric tube during the construction of the test model.

**Unsteady pumping test theory with circular constant water head boundary of finite scale**
According to the derived and transformed theory of constant flow rate unsteady flow pumping test in a fully penetrating well in confined aquifer with circular constant water head boundary of finite scale, we select the data of #3 and #8 piezometers tube with r/b of 0.3 and the data of #4 and #7 piezometers tube with r/b of 0.5. The transmissivity coefficient and storage coefficient at the positions of 3#, 4#, 7# and 8# piezometric tubes are calculated by the proposed method. The matching diagrams of parameter calculation based on the data of 3# and 4# piezometric tubes in the second and third groups of tests are shown in Figure 5.

The calculation results of transmissivity and storage coefficients by the proposed method are shown in Table 7 and Table 8, respectively. All the calculated values based on the proposed method are similar with those of steady flow. The calculated values based on the data of other piezometric tubes by the proposed method are similar to the value of transmissivity coefficients of the geotextile test except for 8# piezometric tube, which shows that the piezometric tubes are indeed affected by the circular constant water head boundary when carrying out the unsteady pumping test in the laboratory of finite scale. In addition, it can also verify the applicability and accuracy of calculating the permeability parameters according to the proposed method.

The storage coefficients calculated by the proposed method are basically similar to that of the conventional unsteady flow pumping test Theis theory, which indicates that the influence of circular constant water head boundary of finite scale on the storage coefficient is not significant.

According to calculation results in Table 4, Table 5 and Table 7, it can be seen that the calculation results based on #8 piezometric tube data by the same theoretical method are basically the same in the three groups of pumping tests with different flow rates. However, Table 4 and Table 7 also show that the calculation results based on #8 piezometric tube data are larger in the three groups of pumping tests with different flow rates, which indicates that the high value is not the test operation error rather than the high hydraulic conductivity near the position of the 8# piezometer tube. Secondly, the influence of boundary can be excluded. Because the calculation results in Table 4 are calculated according to the steady flow Dupuit theory, which is not affected by the finite scale boundary. Moreover, if #8 piezometric tube data are affected by the boundary, the calculation results based on #7 piezometric tube data in Table 4 and Table 7 should be larger than that based on #8 piezometric tube data. The position of #7 piezometer tube is closer to the constant water head boundary than #8 piezometer tube, and the position of #7 piezometer tube should be more affected by the boundary. Combined with the results of geotextile parameters test and slug test, it can also be proved that the calculation results based on #8 piezometric tube data are larger. Therefore, the calculation results based on #8 piezometric tube data are larger, which is caused by the
high hydraulic conductivity at the position of #8 piezometer tube due to improper operation in the process of model construction.

**SLUG TEST**

In order to further verify the accuracy and reliability of the proposed method in this work, the slug test method of determining the permeability parameters of rock and soil with a small test influence radius (within 10 times of the diameter of the test hole) was applied. The permeability of soil can be determined by packing fraction (Wang et al. 2018a; Wang et al. 2019; Zheng et al. 2019a) of the soil mass and the soil particle distribution (Schubert et al. 2019; Zhang et al. 2019). Both of these two parameters are also important to the permeability resistance (Yu et al. 2020) and mechanical properties of the soil (Zheng et al. 2014; Zheng et al. 2019b).

The data collected in slug test is calculated by the theory proposed by Kipp Jr. (1985), Zhou et al. (2016) and Zhao and Zhou (2010), which includes four steps. Firstly, the logarithm of time is taken to draw the measured curve of the water level change \( w \) and time \( t \) in the test borehole on the semi-logarithmic paper with the same modulus as the standard curve, according to the corresponding observation data of the water level change \( w \) and time \( t \) in the borehole. Then, make the coordinate origin of the measured curve and the standard curve at the same height on the semi logarithmic paper, and match the two curves by translation of the time coordinate axis \( t \) of the former. Thirdly, record the relative damping coefficient \( \zeta \) and dimensionless storage coefficient \( \alpha \) of the matched standard curve. Fourthly, select a matching point, record the \( w' \) and \( \hat{t} \) value of it on the standard curve coordinate, and record the water level change \( w \) and time \( t \) value of it on the measured curve coordinate at the same time. At last, calculate the storage coefficient and transmissivity coefficient according to the formula (12) and (13):

\[
S = \left( r_c^2 \right) / \left( 2 r_s^2 \alpha \right) \tag{12}
\]

\[
T = \frac{\beta \cdot \hat{t} \cdot r_c^3 \cdot S}{16 \cdot \zeta \cdot \hat{t}} \tag{13}
\]

Where, \( S \) is storage coefficient [dimensionless]; \( r_c \) is the well casing radius [L]; \( r_s \) is the screen pipe radius [L]; \( \alpha \) is dimensionless storage coefficient [dimensionless]; \( T \) is transmissivity coefficient [L²T⁻¹]; \( \beta \) is dimensionless coefficient of inertia [dimensionless]; \( \zeta \) is damping coefficient [dimensionless]; \( \hat{t} \) is the time [T]; \( \hat{t} \) is the dimensionless time [dimensionless].

Four groups of water injection slug tests (Test No.: 10001-01, 10001-02, 10001-03, 10001-04) and five groups of air pressure slug tests (Test No.: 10002-01, 10002-02, 10002-03, 10002-04, 10002-05) were carried out.
Among them, the diagrams based on the matching curve of No. 10001-01 in water injection slug test and No. 10002-01 in air pressure slug test are shown in Figure 6.

The calculation results of transmissivity coefficient of slug test are shown in Table 9, which indicate that the water injection and air pressure slug test under different excitation modes are in good consistency. The slug test has good stability in determining the transmissivity of rock and soil mass. The calculation results by data from geotextile parameters test, steady flow pumping test and the proposed method are consistent with each other except the conventional unsteady flow Theis theory, which show the limitations and shortcomings of the application of conventional unsteady flow Theis theory under the condition of circular constant water head boundary of finite scale. The proposed method is accurate and applicable for this particular test condition.

**Discussion**

The results from slug test and geotextile tests can be considered as the references to discuss the accuracy of the hydrogeological parameter calculation by using the methods mentioned above, including the steady flow theory, conventional unsteady flow Theis theory and the proposed method in this work.

The calculated transmissivity coefficients in different methods are shown in Figure 7, as well as the tests results of slug test and geotextile test. The transmissivity coefficients obtained from the proposed method achieve a high level of consistency with the steady flow theory. Moreover, the results from slug test and geotextile test are also consistent with the calculation results by the proposed method based on the data from piezometric tubes. The calculation results by conventional unsteady flow Theis theory based on the data from 3# and 8# piezometric tubes are close to other methods. However, a clear offset shows when using the data from 4# and 7# piezometric tubes with conventional unsteady flow Theis theory to calculate the transmissivity coefficient, which can be concluded that the error is positively correlated to the distance between the piezometric tube and the circular constant water head boundary. The results calculated by the proposed method are compared with those based on geotextile parameters test, steady flow pumping test and slug test. Through comparative study, the validation of the proposed method is confirmed.

The error rates of calculation result by conventional unsteady flow Theis theory and the proposed methods with the data from different piezometric tubes are shown in Figure 8. Here, the error rate $\sigma$ is defined as:

$$\sigma = \left| \left| \frac{T_u (T_{uc}) - T_g}{T_g} \right| \right|$$  

(14)

Where, $T_u$ is the calculated value of transmissivity coefficient by conventional unsteady flow Theis theory; $T_{uc}$ is the calculated value of transmissivity coefficient by the proposed method; $T_g$ is the calculated value of transmissivity coefficient by steady flow theory.
As shown in Figure 8, the error rate of the calculation results by conventional unsteady flow Theis theory is much bigger than that of the proposed method. Most of the error rate of the calculation results by the proposed method are less than 10%. In contrast, the error rate reaches as high as 150% when using the conventional unsteady flow Theis theory to calculate the transmissivity coefficient for the indoor finite scale model. Hence, it shows that the proposed method can correct the error caused by the circular constant water head boundary of finite scale, which also verifies the applicability and accuracy of the proposed method.

The proposed method not only saves a lot of time than the steady flow pumping test, but also can obtain the important hydrogeological parameter of storage coefficient, which cannot be obtained by the steady flow pumping test. Meantime, the proposed method is more accurate than the unsteady flow pumping test on conventional Theis theory under the finite scale boundary conditions, and further improves the application scope of unsteady flow pumping test.

Conclusions

In this research, a new calculation method of hydrogeological parameters of transmissivity and storage coefficients is proposed by the unsteady pumping test theory with circular constant water head boundary of finite scale. The new standard curve of the constant flow rate unsteady flow pumping test theory with circular constant water head boundary of finite scale is proposed, and the specific matching method are introduced in detail. Under the condition of circular constant water head boundary of finite scale, especially under the indoor model condition when using the unsteady flow pumping test data to calculate the permeability parameters, it is found that the results based on conventional unsteady flow Theis theory are obviously larger than the results based on geotextile parameters test, steady flow pumping test and slug test. However, the results based on the proposed method are basically similar with other test results. The proposed method can fight off the effect of circular constant water head boundary of finite scale on unsteady pumping test results. The proposed method also improves and develops the method of laboratory parameter tests, which is conducive to the research of a new theory, method or law in the field of hydrogeology. Usually, when a new theory, method or law is proposed in the field of hydrogeology, the corresponding indoor models need to be constructed for the experimental verification. In order to verify the reliability and accuracy of a new theory, method or law, it is necessary to compare the results based on the new theory, method or law with those of unsteady flow pumping test. If the verification is still based on the conventional unsteady flow Theis theory, some errors will occur. As the proposed method can provide more accurate and reliable comparison results, it is of great significance for the study of hydrogeological parameters testing in indoor model with finite scale. Meanwhile, the proposed method can also be applied to large-scale
circular constant head boundary, such as the study of pumping test in circular terrain, which can provide a good basis for foundation pit dewatering and underground engineering construction in circular terrain.

**Author Contributions:** Conceptualization, Y.Z.; methodology, Y.Z.; validation, Z.Z., R.R. and X.D.; formal analysis, Y.Z.; resources, Y.Z.; data curation, Y.Z.; writing—original draft preparation, Y.Z.; writing—review and editing, Z.Z.; visualization, R.R.; supervision, Y.Z., J.W.; project administration, Y.Z., J.W.; funding acquisition, Y.Z. All authors have read and agreed to the published version of the manuscript.

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**Conflicts of Interest:** The authors declare no conflict of interest.

**References**


Fig. 1. The physical model: (a) The plane diagram of the test model; (b) The sectional diagram of the test model; (c) The physical picture of the cement cylindrical base and the pressure box; (d) The overall physical picture of the test model. The blue scale bars in (c) and (d) indicate 0.5 m.
Fig. 2. The proposed method standard curve of $F(\tau, r) - \tau_1$. 
Fig. 3. The thickness test of geotextile: (a) Testing process of Geotextile Thickness; (b) The relationship between thickness and applied stress of the geotextile used in this work.
Fig. 4. The horizontal transmissivity coefficients test of geotextile: (a) Test diagram of horizontal permeability coefficient of Geotextile (after JTJ/T 1999); (b) The horizontal transmissivity coefficients curve of the applied geotextile.
Fig. 5. Transmissivity and storage coefficients matching diagram by the proposed method: (a) 3# piezometric tube of No.10000-02; (b) 4# piezometric tube of No.10000-02; (c) 3# piezometric tube of No.10000-03; (d) 4# piezometric tube of No.10000-03 (Red circle represents the measured data and red cross represents the matching point).
Fig. 6. Matching curve of slug test: (a) Water injection slug test No. 10001-01; (b) Air pressure slug test No. 10002-01 (Red circle represents the measured data).
Fig. 7. Comparison diagram of calculated results by different methods: (a) No.10000-01; (b) No.10000-02; (c) No.10000-03.
Fig. 8. Comparative and analysis diagram of error rate of pumping test: (a) No.10000-01; (b) No.10000-02; (c) No.10000-03.
<table>
<thead>
<tr>
<th>$\tilde{r}$ = $\frac{r}{b}$</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1E-6</td>
<td>0.949</td>
<td>0.547</td>
<td>0.3714</td>
<td>0.113</td>
<td>0.036</td>
<td>0.002</td>
</tr>
<tr>
<td>0.1</td>
<td>0.894</td>
<td>0.534</td>
<td>0.367</td>
<td>0.112</td>
<td>0.035</td>
<td>0.002</td>
</tr>
<tr>
<td>0.3</td>
<td>0.593</td>
<td>0.448</td>
<td>0.318</td>
<td>0.089</td>
<td>0.031</td>
<td>0.002</td>
</tr>
<tr>
<td>0.5</td>
<td>0.346</td>
<td>0.310</td>
<td>0.239</td>
<td>0.076</td>
<td>0.024</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 1 Values of $F_1(\tilde{r}, r)$

$r_1 = \frac{at}{b^2}$

$\tilde{r} = \frac{r}{b}$
Table 2 Values of $F(\bar{r}, r)$

<table>
<thead>
<tr>
<th>$r_1 = \frac{at}{b^2}$</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.515</td>
<td>1.235</td>
<td>1.569</td>
<td>2.079</td>
<td>2.233</td>
<td>2.299</td>
</tr>
<tr>
<td>0.3</td>
<td>0.018</td>
<td>0.308</td>
<td>0.568</td>
<td>1.026</td>
<td>1.142</td>
<td>1.200</td>
</tr>
<tr>
<td>0.5</td>
<td>0.001</td>
<td>0.073</td>
<td>0.215</td>
<td>0.541</td>
<td>0.645</td>
<td>0.691</td>
</tr>
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</table>
### Table 3  Related parameter of vibrating string pressure boxes and stress calculation value

<table>
<thead>
<tr>
<th>Test No.</th>
<th>The sensitivity coefficient ( \times 10^{-4} kPa/Hz^2 )</th>
<th>Initial frequency ( Hz )</th>
<th>Working frequency ( Hz )</th>
<th>Stress value ( kPa )</th>
<th>Average value ( kPa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>02639</td>
<td>1.56</td>
<td>1028</td>
<td>1073</td>
<td>14.7</td>
<td></td>
</tr>
<tr>
<td>02654</td>
<td>1.66</td>
<td>989</td>
<td>1033.1</td>
<td>14.8</td>
<td>14.75</td>
</tr>
</tbody>
</table>
Table 4 Transmissivity coefficient calculated based on steady flow Dupuit formula \((\times 10^{-5} \text{ m}^2/\text{s})\)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>3#</th>
<th>4#</th>
<th>7#</th>
<th>8#</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000-01</td>
<td>1.5</td>
<td>1.32</td>
<td>1.45</td>
<td>2.26</td>
<td>1.63</td>
</tr>
<tr>
<td>10000-02</td>
<td>1.38</td>
<td>1.17</td>
<td>1.25</td>
<td>2.05</td>
<td>1.46</td>
</tr>
<tr>
<td>10000-03</td>
<td>1.59</td>
<td>1.45</td>
<td>1.56</td>
<td>2.32</td>
<td>1.73</td>
</tr>
</tbody>
</table>
Table 5 Transmissivity coefficient calculated by pumping test ($\times 10^{-5}\mbox{ m}^2 \mbox{ s}^{-1}$)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>3#</th>
<th>4#</th>
<th>7#</th>
<th>8#</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000-01</td>
<td>1.67</td>
<td>2.59</td>
<td>2.92</td>
<td>2.41</td>
<td>2.40</td>
</tr>
<tr>
<td>10000-02</td>
<td>1.94</td>
<td>3.12</td>
<td>2.98</td>
<td>2.67</td>
<td>2.68</td>
</tr>
<tr>
<td>10000-03</td>
<td>2.04</td>
<td>2.97</td>
<td>3.04</td>
<td>2.91</td>
<td>2.74</td>
</tr>
</tbody>
</table>
Table 6 Storage coefficient calculated by pumping test ($\times 10^{-3}$)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>3#</th>
<th>4#</th>
<th>7#</th>
<th>8#</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000-01</td>
<td>7.75</td>
<td>5.05</td>
<td>5.18</td>
<td>11.7</td>
<td>7.42</td>
</tr>
<tr>
<td>10000-02</td>
<td>5.41</td>
<td>3.76</td>
<td>3.83</td>
<td>8.55</td>
<td>5.39</td>
</tr>
<tr>
<td>10000-03</td>
<td>5.43</td>
<td>4.2</td>
<td>3.99</td>
<td>8.32</td>
<td>5.49</td>
</tr>
</tbody>
</table>
Table 7 Statistical table of transmissivity coefficient based on the proposed method ($\times 10^{-2} m^2/s$)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>3#</th>
<th>4#</th>
<th>7#</th>
<th>8#</th>
<th>Average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000-01</td>
<td>1.49</td>
<td>1.49</td>
<td>1.41</td>
<td>2.39</td>
<td>1.70</td>
</tr>
<tr>
<td>10000-02</td>
<td>1.57</td>
<td>1.24</td>
<td>1.39</td>
<td>2.03</td>
<td>1.56</td>
</tr>
<tr>
<td>10000-03</td>
<td>1.53</td>
<td>1.38</td>
<td>1.55</td>
<td>2.39</td>
<td>1.71</td>
</tr>
<tr>
<td>Test No.</td>
<td>3#</td>
<td>4#</td>
<td>7#</td>
<td>8#</td>
<td>Average value</td>
</tr>
<tr>
<td>-----------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----------------</td>
</tr>
<tr>
<td>10000-01</td>
<td>12.2</td>
<td>7.38</td>
<td>6.63</td>
<td>16.4</td>
<td>10.65</td>
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<tr>
<td>10000-02</td>
<td>6.61</td>
<td>5.29</td>
<td>4.39</td>
<td>10.7</td>
<td>6.75</td>
</tr>
<tr>
<td>10000-03</td>
<td>7.69</td>
<td>5.14</td>
<td>4.11</td>
<td>12.1</td>
<td>7.26</td>
</tr>
</tbody>
</table>

**Table 8** Statistical table of storage coefficient based on the proposed method ($\times 10^{-3}$)
Table 9 Statistical table of the transmissivity coefficient calculation results of slug test \( \times 10^3 \text{ m}^2 / \text{s} \)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
<th>Average value</th>
<th>Multi group average value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10001-</td>
<td>1.68</td>
<td>1.69</td>
<td>1.58</td>
<td>1.95</td>
<td>-</td>
<td>1.73</td>
<td>1.74</td>
</tr>
<tr>
<td>10002-</td>
<td>1.69</td>
<td>1.67</td>
<td>1.81</td>
<td>1.84</td>
<td>1.75</td>
<td>1.75</td>
<td></td>
</tr>
</tbody>
</table>